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Uniform Physical Theory of Diffraction Equivalent Edge Currents for Truncated Wedge Strips

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Abstract—New uniform closed-form expressions for physical theory of diffraction equivalent edge currents are derived for truncated incremental wedge strips. In contrast to previously reported expressions, the new expressions are well behaved for all directions of incidence and observation and take a finite value for zero strip length. This means that the expressions are well suited for implementation in general computer codes. The new expressions are expressed as the difference between two terms. The first term is obtained by integrating the exact fringe wave current on a wedge along an untruncated incremental strip extending from the leading edge of the structure under consideration. The second term is calculated from an integration of the asymptotic fringe wave (FW) current along another untruncated incremental strip extending from the trailing edge of the structure. The new expressions are tested numerically on a triangular cylinder and the results are compared with those obtained using the method of moments and the previously reported expressions.

I. INTRODUCTION

IN the past decade, a lot of work has been done to develop general computer codes for scattering calculations. In particular, numerous codes have been written for calculating high-frequency bistatic radar scattering from three-dimensional perfectly conducting structures. Some of the original codes were based on the physical optics (PO) approximation. Today, the general codes are required to calculate the scattered field more accurately than what can be achieved using PO. A more accurate calculation can be achieved by adding to the PO field the fringe wave (FW) field which takes into account the diffraction caused by edges. Within the framework of the physical theory of diffraction [1], an approximation to the FW field can be calculated from a line integral along the illuminated part of the edges of the structure by employing one of the closely related approaches known as elementary edge waves [2], incremental length diffraction coefficients [3], or equivalent edge currents [4], [5], [6]. In the present paper, Michaeli's physical theory of diffraction equivalent edge currents [6] will be considered, and these will be referred to as EEC's. The EEC's are determined from an analytical integration of the FW current (the exact current minus the PO current) along incremental strips on the canonical wedge or half-plane.

Closed-form expressions for EEC's have been derived for untruncated (infinite) incremental wedge strips by Ufimtsev [2], Mitzner [3], Michaeli [6], and by Shore and Yaghjian [7], [8];¹ these EEC's will be called *untruncated* EEC's in this paper. For the analysis of bistatic radar scattering there are two problems associated with the untruncated EEC's, namely the presence of the Ufimtsev singularity [6] and the discontinuities of the calculated FW field across the current layers associated with the untruncated strips. The Ufimtsev singularity occurs when the direction of observation is the continuation of an incident field grazing a face of the structure.

The above-mentioned problems associated with the untruncated EEC's are eliminated by using truncated (finite) strips leading to *truncated* EEC's. Closed-form expressions for truncated EEC's have been derived for the half-plane by Breinbjerg [9] and by Shore and Yaghjian [10]. Cote *et al.* [11] have implicitly derived truncated EEC's for a right-angled wedge. Michaeli [12] seems to be the only one who has derived truncated EEC's for a wedge with arbitrary angle. These EEC's apply to the analysis of bistatic radar scattering from three-dimensional structures with flat faces. However, from theoretical considerations, as well as numerical calculations, it appears that Michaeli's truncated EEC's contain nonremovable singularities which are caused by the mathematical procedure applied to obtain closed-form expressions [13]. The singularities occur for various directions of incidence and observation and for zero strip length.

Discontinuities and nonremovable singularities in EEC's employed in general computer codes are unwanted for two reasons. First, the prediction of the scattered field for directions of observation close to discontinuities and nonremovable singularities is clearly inaccurate. Second, the nonremovable singularities give rise to numerical problems when performing the line integration along the edges of the structure. Although the nonremovable singularities usually are confined to a narrow angular region of observation, they do constitute a problem in applications in which the scattered field has to be calculated for all directions of observation. This is the case, for instance, in a power calculation. For those reasons, the untruncated EEC's and Michaeli's truncated EEC's are not well suited for implementation in general computer codes.

In this paper new truncated EEC's are derived. These EEC's do not have the above-mentioned singularity problems of the previously reported expressions, that is, they are well behaved

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¹Only the result by Michaeli [6] is expressed directly in terms of EEC's. The results in [2], [3], and [7] can easily be put in the form of EEC's.

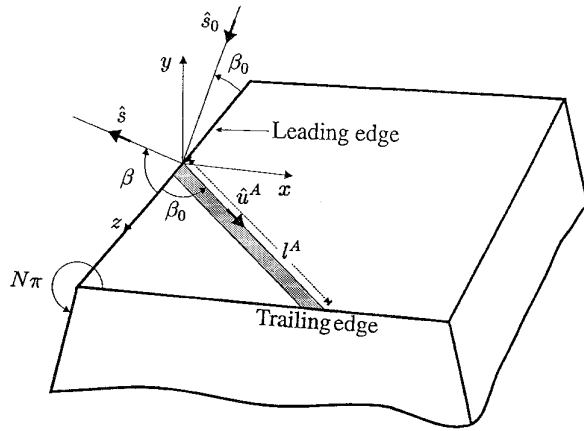


Fig. 1. Three-dimensional view of a flat face of a three-dimensional structure. The truncated incremental strip extends from the leading edge to the trailing edge and is directed along the unit vector \hat{u}^A . The directions of incidence and observation are \hat{s}_0 and \hat{s} , respectively. $N\pi$ is the exterior wedge angle, and it is assumed that $1 < N \leq 2$.

for all directions of incidence and observation and they take a finite value for zero strip length. This means that the new truncated EEC's are, to the knowledge of the author, the first EEC's that are well suited for implementation in general computer codes.

The paper is organized as follows. In Section II, the concept of truncated EEC's is summarized and Michaeli's truncated EEC's are discussed. The derivations leading to the new truncated EEC's are performed in Section III, and Section IV presents numerical examples to illustrate the differences among the fields calculated from the method of moments, the untruncated EEC's, Michaeli's truncated EEC's, and the new truncated EEC's. Concluding remarks are made in Section V. This paper is a condensed version of a Rome Laboratory in-house report [14].

II. THE CONCEPT OF TRUNCATED EEC'S

The configuration under consideration is a perfectly conducting three-dimensional structure with flat faces illuminated by a plane wave (see Fig. 1). In the far field of the structure, a high-frequency approximation to the FW field is calculated from a line integral along the illuminated part C of the edges of the structure. The truncated EEC's are represented by the magnetic current M_T and the electric current I_T , so that the electric FW field is given by [4]

$$\vec{E}^{fw} \sim jk \int_C (ZI_T \hat{s} \times (\hat{s} \times \hat{t}) + M_T \hat{s} \times \hat{t}) \frac{\exp(-jks)}{4\pi s} dC. \quad (1)$$

Herein, j is the imaginary unit (the time factor $\exp(j\omega t)$ is suppressed), k is the wave number, Z is the intrinsic impedance of the ambient medium, $\vec{s} = \hat{s}s$ is the vector to the far-field observation point, and \hat{t} is the edge unit tangent vector. The two adjoining faces at each edge are denoted by A and B . Introducing a local rectangular xyz system at the integration point with $\hat{z} = \hat{t}$ and \hat{y} being the outward normal unit vector of face A , \hat{s} is expressed

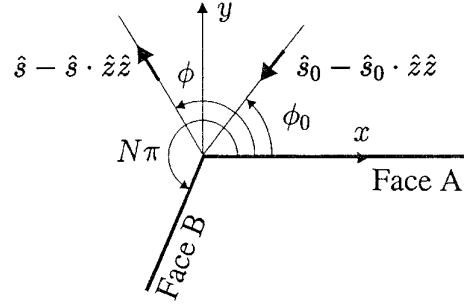


Fig. 2. Two-dimensional view of the configuration shown in Fig. 1 in the plane $z = 0$.

as $\hat{s} = \hat{x} \sin \beta \cos \phi + \hat{y} \sin \beta \sin \phi + \hat{z} \cos \beta$ and the propagation direction \hat{s}_0 of the incident plane wave is $\hat{s}_0 = -\hat{x} \sin \beta_0 \cos \phi_0 - \hat{y} \sin \beta_0 \sin \phi_0 + \hat{z} \cos \beta_0$ (see Figs. 1 and 2). Face B is located in the plane described by $\phi = N\pi$ where $N\pi$ is the exterior wedge angle. Throughout the paper it is assumed that $1 < N \leq 2$.

The truncated EEC's are determined by a sum of two contributions, one from each of the faces A and B

$$M_T = M_T^A + M_T^B \quad \text{and} \quad I_T = I_T^A + I_T^B. \quad (2)$$

Henceforth, the superscripts A and B refer to the contributions from the faces A and B , respectively. In this paper, the contribution from face A will be derived in detail, and the contribution from face B is then obtained from the result for face A using a substitution technique.

The contribution from face A to the truncated EEC's, M_T^A and I_T^A , is calculated analytically by integrating the FW current on face A of a wedge appropriately conforming to the structure along a truncated incremental strip with the length l^A . The strip extends from the *leading edge* (the edge at which the EEC's are placed) to the *trailing edge* and is directed along the unit vector \hat{u}^A which is the intersection of the Keller cone and the face A , as shown in Fig. 1. However, the integration of the exact FW current along the truncated incremental strip cannot be performed exactly in closed form, and thus, an asymptotic calculation is necessary. To this end, the truncated EEC's are expressed as the difference between the untruncated EEC's and the *correction* EEC's

$$M_T = M_{UT} - M_{cor} \quad \text{and} \quad I_T = I_{UT} - I_{cor}. \quad (3)$$

Michaeli [6], [12] found that the untruncated EEC's can be expressed exactly in closed form, whereas closed-form expressions for the correction EEC's can only be obtained using an asymptotic technique. In the present paper, Michaeli's untruncated EEC's are used but a new asymptotic calculation of the correction EEC's is performed because Michaeli's correction EEC's contain nonremovable singularities.

The contribution from face A to the untruncated EEC's, M_{UT}^A and I_{UT}^A , is obtained by integrating the FW current on face A along an untruncated incremental strip. The strip extends from the leading edge and is directed along \hat{u}^A .

Michaeli found that [6, (4)–(7)]

$$M_{UT}^A = \frac{2jZH_{z0} \sin \phi}{k \sin \beta_0 \sin \beta} \left(\frac{U(\pi - \phi_0)}{\mu + \cos \phi_0} + \frac{\sin \frac{\pi - \alpha}{N}}{N \sin \alpha \left(\cos \frac{\pi - \alpha}{N} - \cos \frac{\phi_0}{N} \right)} \right) \quad (4)$$

and

$$I_{UT}^A = \frac{2j}{kN \sin \beta_0 \left(\cos \frac{\pi - \alpha}{N} - \cos \frac{\phi_0}{N} \right)} \cdot \left(\frac{H_{z0} \sin \frac{\pi - \alpha}{N}}{\sin \alpha} (\cot \beta \cos \phi - \mu \cot \beta_0) - \frac{E_{z0} \sin \frac{\phi_0}{N}}{Z \sin \beta_0} \right) + \frac{2jU(\pi - \phi_0)}{k \sin \beta_0 (\mu + \cos \phi_0)} \cdot \left(H_{z0} (\cot \beta \cos \phi + \cot \beta_0 \cos \phi_0) - \frac{E_{z0} \sin \phi_0}{Z \sin \beta_0} \right) - \frac{2j \cot \beta_0 H_{z0}}{kN \sin \beta_0} \quad (5)$$

where H_{z0} and E_{z0} are the z components of the incident magnetic and electric field, respectively, at the origin of the local xyz system, and $U(x)$ is the unit step function. Moreover,

$$\mu = \frac{\sin \beta_0 \sin \beta \cos \phi + \cos \beta_0 (\cos \beta - \cos \beta_0)}{\sin^2 \beta_0} \quad (6)$$

and α is the solution to $\mu = \cos \alpha$ determined by

$$\alpha = -j \operatorname{Log} (\mu + \sqrt{\mu^2 - 1}) \quad (7)$$

with $\operatorname{Log} z = \ln |z| + j \operatorname{Arg} z$ and $-\pi < \operatorname{Arg} z \leq \pi$. The square root in (7) is defined as

$$\sqrt{\mu^2 - 1} = \begin{cases} -|\sqrt{\mu^2 - 1}|, & \mu < -1 \\ j|\sqrt{1 - \mu^2}|, & -1 < \mu < 1. \end{cases} \quad (8)$$

The only nonremovable singularity in M_{UT}^A and I_{UT}^A is the Ufimtsev singularity [6] which occurs when $\mu = 1$ ($\alpha = 0$) and simultaneously, $\phi_0 = \pi$ —that is, when the direction of observation is the continuation of an incident field grazing face A .

The contribution from face A to the correction EEC's, M_{cor}^A and I_{cor}^A , is obtained by integrating the FW current on face A along another untruncated incremental strip. This strip extends from the point of truncation at the trailing edge and is directed

along \hat{u}^A . Michaeli found that M_{cor}^A and I_{cor}^A are calculated from the relations [12, (3)–(7)]

$$M_{\text{cor}}^A = -Z \sin \beta_0 \frac{\sin \phi}{\sin \beta} L_x^A \quad (9)$$

and

$$I_{\text{cor}}^A = \sin \beta_0 (L_z^A - \cot \beta \cos \phi L_x^A) \quad (10)$$

with

$$L_{x,z}^A = \int_{l^A}^{\infty} J_{x,z}^{fw,A} \exp(jku\hat{s} \cdot \hat{u}^A) du \quad (11)$$

where $J_{x,z}^{fw,A}$ denotes the x - and z components of the FW current on face A . The approach used by Michaeli [12] to calculate the integral $L_{x,z}^A$ in (11) is as follows. First, the expressions for the exact FW current, which is given in terms of contour integrals in the complex plane [12, (9)], are inserted into the integral (11). Next, the order of integration is interchanged, the inner integral is calculated analytically and finally, the resulting integral is evaluated asymptotically for $L = kl^A \sin^2 \beta_0 \gg 1$. However, this asymptotic evaluation gives rise to two problems in the correction EEC's when $N \neq 2$. First, the correction EEC's tend to infinity as $L \rightarrow 0$. The quantity L can become small for edge points close to corners in the evaluation of the integral (1). As discussed in [12], it is possible to avoid small values of L by omitting part of the edge which is close to corners. However, this approach is not robust in practical applications because the calculated field will depend on the ratio of the edge being omitted. Second, the correction EEC's contain nonremovable singularities for $\phi_0 = -(\pi + \alpha) + 2\pi N$ and for $\alpha = \pi$, and they are caused by the fact that only few of the poles potentially nearby the saddle point are isolated in the decomposition [12, (29)] applied by Michaeli. For a detailed discussion on these singularities, the reader is referred to [14]. The singularities occur for various directions of incidence and observation. It should be noted that no singularity problems occur in the correction EEC's when $N = 2$.

III. DERIVATION OF NEW CORRECTION EEC'S

In this section, a new approach is used to calculate the correction EEC's. Instead of employing the exact expressions for the FW current when calculating the integral $L_{x,z}^A$ in (11), the asymptotic expressions for the FW current are employed. Thus, the first thing that will be dealt with in this section is the determination of the asymptotic expressions for the FW current.

A. Uniform Asymptotic Expressions for the FW Current on Face A

The x component of the FW current on face A is given by [12, (9)]

$$J_x^{fw,A} = \frac{H_{z0} \exp(-jkz \cos \beta_0)}{j\pi N} \cdot \int_{\Gamma} \frac{\sin \frac{\xi}{N} \exp(jkx \sin \beta_0 \cos \xi) d\xi}{\cos \frac{\xi}{N} - \cos \frac{\phi_0}{N}} \quad (12)$$

where Γ is the steepest descent path through π . Using the substitution

$$s = -\sqrt{\frac{2}{j}} \cos \frac{\xi}{2} \quad (13)$$

where \sqrt{j} means $\exp(j(\pi/4))$, and a decomposition technique similar to the one applied by Michaeli [12, (29)] to isolate the pole potentially nearby the saddle point ($s = 0$), the integrand in (12) is written as a sum of a simple pole term and a regular term so that

$$J_x^{fw,A} = \frac{\sqrt{2}H_{z0} \exp(-jk\hat{u}^A \cdot \vec{r})}{\sqrt{j}\pi N} \cdot \int_{-\infty}^{\infty} \left(\frac{A}{s + \frac{a}{\sqrt{j}}} + R(\xi) \right) \exp(-kx \sin \beta_0 s^2) ds. \quad (14)$$

Herein, $a = \sqrt{2} \cos(\phi_0/2)$, $A = -N/\sqrt{2j}$, $\vec{r} = \hat{x}x + \hat{z}z$, and the regular term is

$$R(\xi) = \frac{\sin \frac{\xi}{N}}{\sin \frac{\xi}{2} \left(\cos \frac{\xi}{N} - \cos \frac{\phi_0}{N} \right)} - \frac{A}{s + \frac{a}{\sqrt{j}}}. \quad (15)$$

The integration of the simple pole term in (14) is performed exactly and expressed in terms of a Fresnel function [15, (12)]. Since the quantity $R(\xi)$ given in (15) is regular near the saddle point, the integration of $R(\xi)$ in (14) is evaluated asymptotically for $kx \sin \beta_0 \gg 1$ using the standard steepest descent technique [16]. The result of these calculations is

$$J_x^{fw,A} \sim -2H_{z0} \exp(-jk\hat{u}^A \cdot \vec{r}) \cdot \left[\text{sign} \left(\cos \frac{\phi_0}{2} \right) F \left(\sqrt{2kx \sin \beta_0} \left| \cos \frac{\phi_0}{2} \right| \right) - \frac{1}{\sqrt{2j\pi kx \sin \beta_0}} \left(\frac{1}{2 \cos \frac{\phi_0}{2}} + \frac{\sin \frac{\pi}{N}}{N \left(\cos \frac{\pi}{N} - \cos \frac{\phi_0}{N} \right)} \right) \right] \quad (16)$$

where F is a modified Fresnel function [12, (37)]

$$F(x) = \sqrt{\frac{j}{\pi}} \exp(jx^2) \int_x^{\infty} \exp(-jt^2) dt. \quad (17)$$

A similar procedure is used to obtain the asymptotic result for $J_z^{fw,A}$ [14]

$$J_z^{fw,A} \sim -2 \exp(-jk\hat{u}^A \cdot \vec{r}) \left[\left(\frac{E_{z0} \sin \phi_0}{Z \sin \beta_0} - H_{z0} \cot \beta_0 \cos \phi_0 \right) \cdot \text{sign} \left(\cos \frac{\phi_0}{2} \right) F \left(\sqrt{2kx \sin \beta_0} \left| \cos \frac{\phi_0}{2} \right| \right) \right]$$

$$- \frac{E_{z0} \sin \phi_0}{Z \sin \beta_0 2\sqrt{2j\pi kx \sin \beta_0} \cos \frac{\phi_0}{2}} + \frac{H_{z0} \cot \beta_0}{\sqrt{2j\pi kx \sin \beta_0}} \cdot \left(\frac{\cos \phi_0}{2 \cos \frac{\phi_0}{2}} - \frac{\sin \frac{\pi}{N}}{N \left(\cos \frac{\pi}{N} - \cos \frac{\phi_0}{N} \right)} \right) \quad (18)$$

which applies for $kx \sin \beta_0 \gg 1$.

B. Expressions for the New Correction EEC's

The asymptotic expressions for the contribution from face A to the correction EEC's, M_{cor}^A in (9) and I_{cor}^A in (10), are now obtained by inserting the expressions (16) and (18) for the FW current into the integral $L_{x,z}^A$ in (11). By using the relations

$$\int_{1^A}^{\infty} \frac{\exp(-jku \sin^2 \beta_0 (1-\mu))}{\sqrt{u}} du = \frac{2\sqrt{\pi} \exp(jL(\mu-1))}{\sqrt{jk(1-\mu)} \sin \beta_0} F(\sqrt{L(1-\mu)}) \quad (19)$$

and

$$\int_{1^A}^{\infty} F \left(\sqrt{2ku \sin \beta_0} \left| \cos \frac{\phi_0}{2} \right| \right) \cdot \exp(-jku \sin^2 \beta_0 (1-\mu)) du = \frac{\exp(jL(\mu-1))}{jk \sin^2 \beta_0 (\mu + \cos \phi_0)} \cdot \left(-F \left(\sqrt{2L} \left| \cos \frac{\phi_0}{2} \right| \right) + \frac{\sqrt{2} \left| \cos \frac{\phi_0}{2} \right|}{\sqrt{1-\mu}} F(\sqrt{L(1-\mu)}) \right) \quad (20)$$

where μ and F are defined in (6) and (17), respectively, and $L = kl^A \sin^2 \beta_0$, the result for the new correction EEC's is obtained through a straightforward calculation [14]

$$M_{\text{cor}}^A \sim \frac{2ZH_{z0} \sin \phi \exp(jL(\mu-1))}{jk \sin \beta \sin \beta_0} \cdot \left[\frac{-\text{sign} \left(\cos \frac{\phi_0}{2} \right)}{\mu + \cos \phi_0} F \left(\sqrt{2L} \left| \cos \frac{\phi_0}{2} \right| \right) + \left(\frac{\sqrt{1-\mu}}{\sqrt{2}(\mu + \cos \phi_0) \cos \frac{\phi_0}{2}} - \frac{\sqrt{2} \sin \frac{\pi}{N}}{N \sqrt{1-\mu} \left(\cos \frac{\pi}{N} - \cos \frac{\phi_0}{N} \right)} \right) \cdot F(\sqrt{L(1-\mu)}) \right] \quad (21)$$

and

$$I_{\text{cor}}^A \sim \frac{2 \exp(jL(\mu-1))}{jk \sin \beta_0 (\mu + \cos \phi_0)} \left[\text{sign} \left(\cos \frac{\phi_0}{2} \right) \cdot \left(\frac{E_{z0} \sin \phi_0}{Z \sin \beta_0} - H_{z0} (\cot \beta_0 \cos \phi_0 + \cot \beta \cos \phi) \right) \cdot F \left(\sqrt{2L} \left| \cos \frac{\phi_0}{2} \right| \right) + \sqrt{2(1-\mu)} \cdot \left(-\frac{E_{z0} \sin \frac{\phi_0}{2}}{Z \sin \beta_0} + \frac{H_{z0}}{2 \cos \frac{\phi_0}{2}} \right) \cdot (\cot \beta_0 \cos \phi_0 + \cot \beta \cos \phi) \right. \\ \left. + \frac{H_{z0} \sin \frac{\pi}{N} (\mu + \cos \phi_0) (\cot \beta_0 - \cot \beta \cos \phi)}{N \left(\cos \frac{\pi}{N} - \cos \frac{\phi_0}{N} \right) (1-\mu)} \right] \cdot F(\sqrt{L(1-\mu)}) \quad (22)$$

These asymptotic expressions apply for $L \gg 1$. The final expressions for the truncated EEC's are obtained by first calculating M_T^A and I_T^A by subtracting the above results (21), (22) from the untruncated EEC's (4), (5), as shown in (3). Second, the contribution from face B is calculated using the results for M_T^A and I_T^A by replacing β_0 with $\pi - \beta_0$, β with $\pi - \beta$, ϕ_0 with $N\pi - \phi_0$, ϕ with $N\pi - \phi$, and l^A with l^B . Third, the contributions from the two faces are added to determine M_T and I_T , see (2). Finally, these expressions are inserted into the radiation integral (1) to determine the approximate FW field from the truncated EEC's.

It is noted that M_{cor}^A in (21) and I_{cor}^A in (22) do not contain singularities for $\alpha = \pi$ ($\mu = -1$), $\phi_0 = -(\pi + \alpha) + 2\pi N$, and $L = 0$ as do the previously reported expressions [12]. Using the result of [6, Appendix II] it is shown that if $\phi_0 \neq \pi$, M_{cor}^A and I_{cor}^A remain bounded as $\mu \rightarrow 1$. If $\phi_0 = \pi$ and $\mu \rightarrow 1$, M_{cor}^A and I_{cor}^A are singular but this singularity (the Ufimtsev singularity) is cancelled by the singularity in M_{UT}^A and I_{UT}^A given by (4) and (5), respectively. This means that M_T and I_T are valid for all directions of incidence and observation. Furthermore, the fact that M_{cor}^A and I_{cor}^A are finite for $L = 0$ implies that no numerical problems arise for edge points close to corners when evaluating the integral (1). This is very convenient from a practical point of view. However, the field calculated from the truncated EEC's for edge points close to corners is a poor approximation to the exact field because no information on the distortion of the current near corners [17] is introduced.

C. Special Cases

If it is assumed that the arguments $\sqrt{2L} |\cos(\phi_0/2)|$ and $\sqrt{L(1-\mu)}$ of the modified Fresnel functions in (21) and (22) are so large that the asymptotic formula [12, (40)]

$$F(x) \sim \frac{1}{2x\sqrt{j\pi}} \quad (23)$$

can be applied, M_{cor}^A in (21) and I_{cor}^A in (22) become

$$M_{\text{cor}}^A \sim \frac{-\sqrt{2} Z H_{z0} \sin \phi \sin \frac{\pi}{N} \exp(jL(\mu-1))}{jk \sin \beta \sin \beta_0 \sqrt{j\pi L N} (1-\mu) \left(\cos \frac{\pi}{N} - \cos \frac{\phi_0}{N} \right)} \quad (24)$$

and

$$I_{\text{cor}}^A \sim \frac{\sqrt{2} H_{z0} \sin \frac{\pi}{N} (\cot \beta_0 - \cot \beta \cos \phi) \exp(jL(\mu-1))}{jk \sin \beta_0 \sqrt{j\pi L N} (1-\mu) \left(\cos \frac{\pi}{N} - \cos \frac{\phi_0}{N} \right)} \quad (25)$$

Except for $\phi_0 = -(\pi + \alpha) + 2\pi N$ and $\alpha = \pi$ ($\mu = -1$) these expressions are the same as those obtained using Michaeli's correction EEC's [12].

For the half-plane, i.e., $N = 2$, the expressions for M_{cor}^A in (21) and I_{cor}^A in (22) simplify. Besides, in this case the uniform asymptotic FW current found in Section III-A equals the exact FW current. This means that M_{cor}^A in (21) and I_{cor}^A in (22) are valid for any value of L . The correction EEC's, that is, the sum of the contributions from faces A and B , become

$$M_{\text{cor}} = \frac{4Z H_{z0} \sin \phi \exp(jL(\mu-1))}{jk \sin \beta \sin \beta_0 (\mu + \cos \phi_0)} \left(-\text{sign} \left(\cos \frac{\phi_0}{2} \right) \cdot F \left(\sqrt{2L} \left| \cos \frac{\phi_0}{2} \right| \right) + \frac{\sqrt{2} \cos \frac{\phi_0}{2}}{\sqrt{1-\mu}} \cdot F(\sqrt{L(1-\mu)}) \right) \quad (26)$$

and

$$I_{\text{cor}} = \frac{4E_{z0} \sin \frac{\phi_0}{2} \exp(jL(\mu-1))}{jk Z \sin^2 \beta_0 (\mu + \cos \phi_0)} \left(2 \left| \cos \frac{\phi_0}{2} \right| \cdot F \left(\sqrt{2L} \left| \cos \frac{\phi_0}{2} \right| \right) - \sqrt{2(1-\mu)} F(\sqrt{L(1-\mu)}) \right) \\ + \frac{4H_{z0} \exp(jL(\mu-1))}{jk \sin \beta_0 (\mu + \cos \phi_0)} \left(-\text{sign} \left(\cos \frac{\phi_0}{2} \right) \cdot (\cot \beta_0 \cos \phi_0 + \cot \beta \cos \phi) F \left(\sqrt{2L} \left| \cos \frac{\phi_0}{2} \right| \right) \right. \\ \left. + \frac{\sqrt{2} \cos \frac{\phi_0}{2} (\cot \beta \cos \phi - \mu \cot \beta_0)}{\sqrt{1-\mu}} \cdot F(\sqrt{L(1-\mu)}) \right) \quad (27)$$

which are the same results obtained using Michaeli's expressions [12]. The expressions (26) and (27) are further verified by letting $l^A = 0$. In this case $M_{\text{cor}} = M_{UT}$ and $I_{\text{cor}} = I_{UT}$. Thus, the truncated EEC's, M_T and I_T in (3), are zero which is the result obtained by integrating the FW current along a strip with length zero.

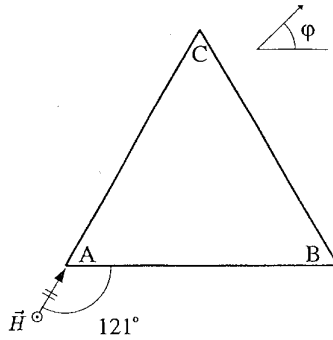


Fig. 3. Cross-section of triangular cylinder with side length 2λ illuminated by a TE plane wave.

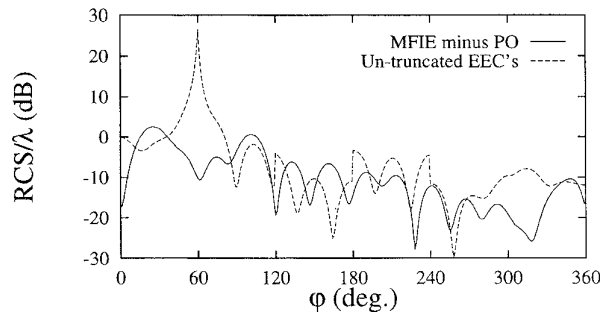


Fig. 4. FW field for the configuration shown in Fig. 3.

IV. NUMERICAL RESULTS

In this section, the FW field scattered by a two-dimensional perfectly conducting triangular cylinder is calculated using both the EEC's and the method of moments applied to the magnetic field-integral equation (MFIE). The purpose of the numerical calculations is to illustrate that the new truncated EEC's do not predict infinities in the far field as do both the untruncated EEC's and the previously reported truncated EEC's [12].

The lengths of the three sides of the triangular cylinder are all equal to 2λ , λ being the wavelength, and the illuminating field is a TE-polarized plane wave with direction of incidence shown in Fig. 3. The FW field is calculated in the far field of the structure and expressed in terms of the two-dimensional radar cross section (RCS). The direction to the far-field observation point is determined by the angle ϕ . Fig. 4 shows the FW field calculated from the difference between the MFIE solution and the PO solution, and from the untruncated EEC's. It is seen that the untruncated EEC's yield a poor approximation to the exact scattered FW field: the Ufimtsev singularity occurs for $\phi = 60^\circ$ and the field is discontinuous across the current layers located at $\phi = 120^\circ, 180^\circ, 240^\circ$, and 360° . In addition, discontinuities at $\phi = 60^\circ$ and $\phi = 300^\circ$ exist but these cannot be seen on the RCS plot of Fig. 4. However, the phase of the scattered field reveals the discontinuities.

The Figs. 5 and 6 show the results when the truncated EEC's are used. Fig. 5 shows the results obtained from the previously reported truncated EEC's and in Fig. 6 the results obtained from the new truncated EEC's are shown. From both figures

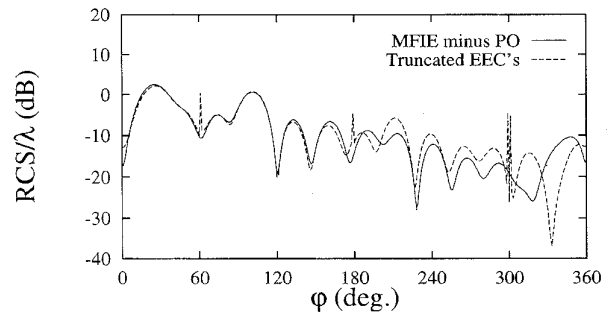


Fig. 5. FW field for the configuration shown in Fig. 3. The previously reported expressions [10] are used to calculate the truncated EEC's.

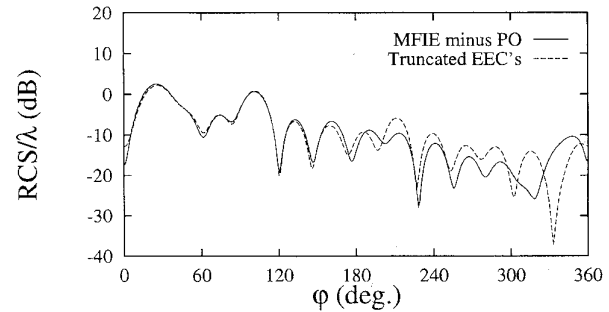


Fig. 6. FW field for the configuration shown in Fig. 3. The new truncated EEC's are employed.

it is seen that the Ufimtsev singularity and the discontinuities across the current layers disappear. However, Fig. 5 reveals that five spikes occur in the far field obtained from the previously reported truncated EEC's when $\phi = 61, 179, 299, 300$, and 301° . These spikes are caused by the nonremovable singularities in the expressions. The spikes at $\phi = 61, 179, 299$, and 301° are caused by the singularity occurring when $\phi_0 = -(\alpha + \pi) + 2\pi N$ (see the discussion in Section II) which is almost satisfied for edges *B* and *C* (see Fig. 3). The spike at $\phi = 300$ occurs because α is close to π (see Section II) for edge *B*. Although the singularities only affect a small angular region, they constitute a problem in practical applications in which the far field has to be predicted for directions of observation close to the singularities. As noticed from Fig. 6, no spikes occur when the new truncated EEC's are used. However, the agreement between the two methods of calculation is not perfect. The reason for this discrepancy is that the truncated EEC's only take into account part of the second-order edge diffraction because the FW current exited at the trailing edge is neglected.

V. CONCLUSION

New closed-form uniform expressions for physical theory of diffraction equivalent edge currents have been derived for truncated incremental wedge strips. The new truncated EEC's are well behaved for all directions of incidence and observation. The expressions are asymptotic for $L \gg 1$, L being a parameter proportional to the strip length; however, they take a finite value when L is zero. This implies, in contrast

to the previously reported expressions, that the new truncated EEC's are well suited for implementation in general computer codes calculating the bistatic radar scattering from perfectly conducting three-dimensional structures with plane faces.

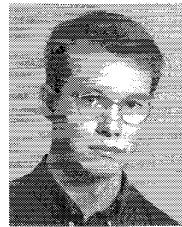
Future work may address the problem of enhancing the accuracy of the truncated EEC's by taking into account the FW current exited at the trailing edge, for instance, by employing the procedure introduced by Breinbjerg [9].

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